Quanto Local Volatility Model

We review a model for computing the price, in the domestic currency, of European standard call and put options on an underlying foreign equity (stock or index) with tenor of up to 7 years. The function implements a local volatility based pricing method.

The payoff in the domestic currency of a European option at maturity T is given by

$$(\psi\,(S_T - K))^+ \,,$$

where

- ψ is +1 for a call option and -1 for a put option,
- S_T is the foreign spot equity level at time T, and
- K is a strike level.

The equity price process satisfies a risk-neutral stochastic differential equation (SDE) when where are no dividend payments. Let St denote the equity price at time t. We assume that the process satisfies a SDE of the form under the domestic risk-neutral probability measure:

$$dS_t = S_t\left[(r_t + \hat{q}_t)\,dt + \sigma(S,t;S_0)dW_t\right],$$

where

- r_t is a deterministic short-interest rate,
- $\hat{q}_t$ is a deterministic quanto-adjustment level,
- $\sigma(S,t;S_0)$ is the equity price local volatility, and
- W_t is a standard Brownian motion.

We note that the volatility σ depends only on time and on the instantaneous value of the state variable S, but does not explicitly depend on W.

The quanto-adjustment q is calculated by

$$\int_0^t \hat{q}_s ds = \rho_{fx}\sigma_{fx}\sigma_B(S_0, t)t,$$

where

- ρ_{fx} is an equity-exchange rate correlation parameter,
- σ_{fx} is a particular exchange rate volatility parameter, and
- $\sigma_B(S_0, t)$ is a Black's implied volatility associated with the at-the-money (ATM) strike level, S_0, and tenor, t, such that market call and put prices c and p satisfy,

  $$c = S_0 N(d_1) - e^{-\int_0^T r_s ds} K N(d_2) \quad \text{and} \quad p = e^{-\int_0^T r_s ds} K N(-d_2) - S_0 N(-d_1),$$

  respectively, where

  $$d_1 = \frac{\ln(S_0/K) + \sigma_B(S_0, t)^2 T/2}{\sigma_B(S_0, t)\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_B(S_0, t)\sqrt{T}.$$

Note that we also implement a second quanto-adjustment technique that is of the form

$$\int_0^t \hat{q}_s ds = \rho_{fx}\sigma_{fx} \int \sigma_B(K, t)\omega_K dK,$$

where w_k is a weight.

Consider the calibration of the local volatility function based on market option prices or, equivalently, market Black's implied volatilities. If there exists a smooth surface of either option price or implied volatility as a function of option strike and maturity, then this surface uniquely determines the local volatility function.

Moreover, an explicit expressions for local volatility is provided. For example, if C denotes the price of a call option on a non-dividend paying stock, with a constant risk-free interest rate, then

$$\sigma(K,T;S_0) = 2\frac{\frac{\partial C}{\partial T} + Kr\frac{\partial C}{\partial K}}{K^2\frac{\partial^2 C}{\partial K^2}}.$$

Local volatility model can also be applied to value callable exotic notes. Callable option is more volatile as callable events make the remaining part of the trade potentially be cancelled as a result of a trigger condition or an exercise option.

In general, local volatility model is very useful tool for pricing equity derivatives. The model requires two interest rate curves. One, called the hedge curve, is specified by an array of curve objects. A curve object is a string identifier representing a previously-created list of n term and continuously compounded zero rate pairs. In the context of a quanto option, the hedge curve input argument consists of a forward curve.

The other curve, called the discount curve, is specified by a list of m term and continuously compounded zero rate pairs.

References:

https://finpricing.com/curveVolList.html